INDIAN LOGIC, SIMILARITY, AND ARTIFICIAL INTELLIGENCE

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Abstract: *The text* Nyāyasūtra *includes an inference referring to similarity. That inference has been addressed from different frameworks, from first-order predicate logic to the contemporary theory of mental models. Without ignoring previous accounts, in this paper, I deal with the inference from a computer system named Non-Axiomatic Reasoning System. The logic of the latter system is Non-Axiomatic Logic. My main point here is that Non-Axiomatic logic, which even includes a similarity copula, can easily work with the inference.*

Keywords: *computer system; inference; Non-Axiomatic Logic;* Nyāyasūtra*; similarity*.

Introduction

The Indian text *Nyāyasūtra* includes an inference related to similarity (see, e.g., Ganeri, 2004). I will refer to that inference below with the acronym NSPP (from the expression '*Nyāyasūtra* Similarity Proof Patter'). The inference can be presented as a five-steps process (see also, e.g., López-Astorga, 2023):

- *Thesis*: it indicates the statement to prove. For example, 'property A is one of the properties of element x' (i.e., Ax).

- *Reason*: it points out what leads to think that the *Thesis* is right. It can be, for instance, 'property B is also one of the properties of element x' (i.e., Bx).

- *Example*: it gives three pieces of data that, if they are taken together, support the *Thesis* (Ax): there is an element similar to x, that element has property A too, and that element has property B as well. The pieces of data can be, for example, 'y is similar to x' ($x \approx y$), 'property A is one of the properties of y' (Ay), and property B is also one of the properties of y' (By).

- *Application*: it confirms the *Reason* (Bx).

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- *Conclusion*: the *Thesis* (Ax) is accepted.

These five steps have been analyzed from several points of view. An account was intended to capture them within the western classical logic (Schayer, 1933). However, that account was questioned because it ignored an essential point of NSPP: elements x and y must be similar (e.g., Ganeri, 2004; López-Astorga, 2023).

Other analysis has been given considering the theory of mental models (e.g., Johnson-Laird, Byrne, & Khemlani, 2024) and the way it explains induction (e.g., Johnson-Laird, 2012). This is an analysis offered from the psychology perspective trying to describe the real manner people think when addressing inferences such as NSPP (it is to be found in works such as López-Astorga, 2023).

There is even an account that, while it follows first-order predicate calculus, it also assumes such different approaches as the previous analysis from the theory of mental models, Ganeri's (2004) explanation, and Carnap's (1936, 1937) reduction concept. The key point of this account is that the relation of similarity between x and y consists of the fact that x and y share one more characteristic, property, or predicate distinct from A and B (based on Ganeri's ideas, it is in López-Astorga, 2023).

My goal here is not to argue against any of these analyses or accounts. I only want to show that there is at least a logic from which NSPP can be applied with no difficulties. That logic is NAL, that is, Non-Axiomatic Logic (e.g., Wang, 2013). Given that NAL is the logic supporting computer system NARS, that is, Non-Axiomatic Reasoning System (e.g., Wang, 2006), I will also claim that NARS has no problem working with NSPP. This task is relevant, as there are well-known theorems that do not seem to be acceptable in NAL (see, e.g., López-Astorga, 2024, for the case of Hintikka's theorem).

The paper will have two sections. In the first one, I will describe the general lines of the analyses and accounts mentioned above. In the second one, I will explain the way NAL and NARS can deal with NSPP.

Some previous analyses and accounts of NSPP

An account is that of Schayer (1933). Basically, Schayer tried to translate NSPP into well-formed formulae of western classical logic. We can say that Schayer links NSPP to an inference with this logical form in first-order predicate calculus:

$$\{\exists x\ Bx, \forall y\ (By \Rightarrow Ay)\} \vdash \exists x\ Ax$$

The strength of logical structures such as this one is their simplicity. Their weakness is that they do not capture the essential characteristic of NSPP: the fact that is based on the similarity between two elements such as x and y (see, e.g., Ganeri, 2004; López-Astorga, 2023).

Trying to relate NSPP to the way people derive conclusions, the theory of mental models (see also, e.g., Johnson-Laird, 2023) has been considered. According to this theory, given a statement, individuals think about the possible situations that could be true if that statement were true. Thus, NSPP has been addressed from Johnson-Laird's (2012) work, where the mental process the theory of mental models assigns to induction is explained (e.g., López-Astorga, 2023). The key to this account is in two possible scenarios:

$$\Diamond[Ax \wedge Bx \wedge Ay \wedge By \wedge (x \approx y)] \wedge \Diamond[\neg Ax \wedge Bx \wedge Ay \wedge By \wedge (x \approx y)]$$

I am using '$\Diamond$' to express 'possibility'. However, the theory of mental models is not a modal logic or understands 'possibility' as habitually in modal logic. While this topic is interesting, I will not dwell on it because I would move away from my aims in the present paper. Besides, to make my point, a clarification of how the theory of mental models understands 'possible' is not necessary (for a discussion about the relations between the theory of mental models and modal logic, see, e.g., Johnson-Laird & Ragni, 2024).

Following this explanation, the mental process underlying NSPP consists of the reflection about the two possibilities above. What NSPP shows is that one might think that the first possibility (i.e., $\Diamond[Ax \wedge Bx \wedge Ay \wedge By \wedge (x \approx y)]$) can be deemed as the most probable scenario. That is what leads to accept Ax as the *Conclusion*. To develop this idea, in López-Astorga (2023) an example is presented. That example refers to properties such as 'to be a rabbit' and 'to like carrots'. According to it, a scenario such as that described in the first possibility should be accepted because the fact that a rabbit likes carrots is more probable than the fact that a rabbit does not like carrots.

This analysis based on the theory of mental models is more complex than my summary here. Nevertheless, my comments so far can suffice to have an idea of what this explanation proposes.

One more account is that of Ganeri (2004). It links NSPP to 'case-based reasoning'. Its context is the real context in which physicians make diagnoses. The example is about two patients. $Patient_1$ ate poisonous mushrooms and had symptoms S_1 and S_2. $Patient_2$ ate poisonous mushrooms as well and has symptom S_1. Therefore, a physician can conclude that $Patient_2$ will have symptom S_2 too. This allows supposing that NSPP is a way to reason that is present even in clinical diagnoses.

López-Astorga (2023) considers Ganeri's (2004) account to reveal the relation of similarity that is missing in Schayer's (1933) explanation. That relation consists of the fact of having one more property (different from A and B) in common. If we assume these equivalences,

$A =_{\mathrm{df}} S_2$
$B =_{\mathrm{df}} S_1$
$C =_{\mathrm{df}}$ to eat poisonous mushrooms
$x =_{\mathrm{df}} Patient_2$
$y =_{\mathrm{df}} Patient_1$

We can think, following López-Astorga (2023), that the possibilities the theory of mental models should attribute to NSPP are these:

$$\Diamond[Ax \wedge Bx \wedge Cx \wedge Ay \wedge By \wedge (x \approx y)] \wedge \Diamond[\neg Ax \wedge Bx \wedge Cx \wedge Ay \wedge By \wedge (x \approx y)]$$

These possibilities give further support to the rejection of the second main conjunct and the preference for the first one (i.e., the preference for $\Diamond[Ax \wedge Bx \wedge Cx \wedge Ay \wedge By \wedge (x \approx y)]$). This is because there is one more property (i.e., C) that both x and y have.

In addition, Ganeri's (2004) idea also enables to complete the inference I have assigned to Schayer's (1933) proposal. Considering C, according to López-Astorga (2023), we can think of an inference such as this one:

$$\{\exists x\, (Bx \wedge Cx),\ \forall y\, [(By \wedge Cy) \Rightarrow Ay)\} \vdash \exists x\, Ax$$

The second premise is interesting because it is equivalent to

$$\forall y\ [By \Rightarrow (Cy \Rightarrow Ay)]$$

Which has the structure of Carnap's (1936) reduction sentences. The first premise is also relevant, since it has the structure of the condition Carnap (1936) requires reduction sentences to fulfill. Following Carnap (1936), the latter formula can be a reduction sentence only when this structure is the case:

$$\{\exists y\ (By \wedge Cy)$$

This explanation by López-Astorga (2023) shows that Ganeri's (2004) approach linking NSPP to case-based reasoning improves both the account given from the theory of mental models and that of Schayer (1933). In the first case, to have one more property more strongly supports the choice of the possibility in which Ax is true. In the second case, the additional property, C, enables to deal with the initial problem in Schayer's (1933) account. The latter property can be deemed as the similarity between elements x and y. Besides, it allows coming to structures such as Carnap's reduction sentences. Although in different contexts and with different goals, those structures were created to relate properties, and to check whether the fact of having a property implies to have other property (for a detailed description of all the accounts and analyses in this section, see López-Astorga, 2023).

What I will do next is not to present objections to the previous accounts and analyses. I will try to show that NSPP is a type of inference that can be made in NAL very easily, that is, that NSPP can be computationally treated from NARS. My aim with this is twofold: to propose one more alternative for the debate and to give an example of the logical machinery necessary to make inferences with the structure of NSPP. As indicated below, that machinery appears to have a problem: it needs to determine the level of demanding regarding the similarity between x and y. However, this is not a significant issue. Computer systems can make that kind of decisions by themselves.

NSPP from NAL

To review how NAL can deal with NSPP, first, we must consider the way a property can be attributed to an element in the former. This is made by means of a copula named 'inheritance copula' ('$\rightarrow$'). If we want to say that element x has property A, we can express that as in (1).

(1) $X \rightarrow A$ (see, e.g., Wang, 2013; Definition 2.2).

Inheritance copula, which is transitive (Wang, 2013; Definition 2.2), provides relations of extension and intension between its terms (in this case, x and A). Those relations are not the habitual ones in logic (see also, e.g., Wang, 2023).

Let X^E, X^I, A^E, and A^I be, respectively, the extension of x, the intension of x, the extension of A, and the intension of A. What (1) means is that $X^E \subseteq A^E$ and $A^I \subseteq X^I$ (Wang, 2013; theorem 2.4; set theory is usually a metalanguage in descriptions of NAL). Nonetheless, the relations between x and A can never be absolutely known. NAL considers an environment characterized by AIKR, that is, the 'Assumption of Insufficient Knowledge and Resources' (see also, e.g., Wang, 2011). For this reason, statements such as (1) need a value between 1 and 0 for frequency ('f'), and a value between 1 and 0 for confidence ('c'). (2) is a more appropriate expression in NAL to indicate Ax.

(2) $X \rightarrow A \langle f, c\rangle$ (Wang, 2013; Definitions 3.3 and 3.8).

The formulae to identify the values of f and c are, respectively, "$f = w^+/w$" and "$c = w/(w + k)$" (Wang, 2013, p. 29; Definition 3.3), where w^+ represents 'positive evidence', w stands for 'total evidence', and k is a constant . In Wang (2013), NAL is described assuming that $k = 1$. However, the value of k can be different as the system can have different 'personalities'. The higher its value is, the harder is to accept conclusions (see, in addition to Wang, 2013, e.g., Wang, 2023).

Let us assume that x indicates 'verb' and A denotes 'word'. Given AIKR, the system cannot know all the words or all the verbs. Let us suppose that it knows 50 words, only 15 of them being verbs. In that scenario, (2) would have to be expressed as follows:

$X \rightarrow A \langle 0.3, 0.98\rangle$

Considering what has been explained in the introduction, in NARS language, whose name is 'Narsese' (e.g., Wang, 2013), the *Thesis*, that is, the statement to prove, is (2). The *Reason*, that is, *Bx*, is (3).

(3) $X \to B \langle f, c \rangle$

As far as the *Example* is concerned, it is easy to express in Narsese both *Ay* and *By*. They are (4) and (5).

(4) $Y \to A \langle f, c \rangle$

(5) $Y \to B \langle f, c \rangle$

What is missing is the information on the similarity between x and y. But NAL also has a way to express similarity. It has a 'similarity copula' too: '$\leftrightarrow$'. The similarity between x and y can be expressed in Narsese as in (6).

(6) $X \leftrightarrow Y \langle f, c \rangle$ (see, e.g., Wang, 2013, Definitions 6.1 and 6.2).

Frequency and confidence are also easy to calculate in the case of similarity statements such as (6). It is only necessary to know the following:

Let $w^+(T)$ and $w(T)$ be, respectively, the positive evidence of term T and the total evidence of that very term. In the case of (6), $w^+(X \leftrightarrow Y) = w^+(X \to Y) = w^+(Y \to X)$, and $w(X \leftrightarrow Y) = w(X \to Y) \cup w(Y \to X)$ (Wang, 2013; Definitions 3.2 and 6.2).

Because the *Application* reveals that (3) is the case, what we must check is whether all this allows NAL, and hence NARS, to come to the *Conclusion*, that is, (2). The premises would be (3), (4), (5), and (6). So, the question is:

$$\{X \to B \langle f, c \rangle, Y \to A \langle f, c \rangle, Y \to B \langle f, c \rangle, X \leftrightarrow Y \langle f, c \rangle\} \vdash X \to A \langle f, c \rangle?$$

The answer to this question is positive. NAL has at least to ways to come to (2) from (3), (4), (5), and (6). I will explain that resorting to only three inference rules in NAL: on the one hand, induction and deduction; on the other hand, analogy.

The first way implies the rules of induction and deduction. It is at a minimum interesting that one of the required rules in this first path to derive (2) is the rule of induction. That is an essential concept in the account based on the theory of mental models described above. The rule of induction enables to make this inference:

$$\{Y \to A \langle f_1, c_1\rangle, Y \to B \langle f_2, c_2\rangle\} \vdash B \to A \langle f_3, c_3\rangle$$

Where $f_3 = f_1$ and $c_3 = (f_2 \times c_1 \times c_2)/(f_2 \times c_1 \times c_2 + k)$ (see, e.g., Wang, 2013; for the formulae, especially Table 4.7).

With the conclusion of the latter inference as the first premise, and (3) as the second premise, it is possible to derive in NAL in turn:

$$\{B \to A \langle f_3, c_3\rangle, X \to B \langle f_4, c_4\rangle\} \vdash X \to A \langle f_5, c_5\rangle$$

If $f_5 = f_3 \times f_4$, and $c_5 = f_3 \times c_3 \times f_4 \times c_4$, this is the application of the rule of deduction in NAL (see, e.g., Wang, 2013; for the formulae, especially Table 4.7).

Therefore, first applying the rule of induction to statements (4) and (5), and then applying the rule of deduction to the statement resulting from the previous application and (3), we can obtain (2). One might think that this would be enough to get the expected conclusion in NSPP. However, there are two problems. First, f_5 and c_5 can have low values, which can cause the system to ignore the statement with those values. Second, as in the case of the explanation Schayer (1933) offered, this process does not consider the similarity between x and y, that is, in Narsese, (6).

These two problems can be solved at once because NAL has a rule of analogy needing statements with the structure of (6). In a real implementation of NARS, not only the rules of induction and deduction would be applied. The rule of analogy would also be applied to make this inference:

$$\{Y \to A \langle f_1, c_1\rangle, X \leftrightarrow Y \langle f_6, c_6\rangle\} \vdash X \to A \langle f_7, c_7\rangle$$

In this case, the formulae are $f_7 = f_1 \times f_6$ and $c_7 = f_6 \times c_1 \times c_6$ (see, e.g., Wang, 2013; for the formulae, especially Table 6.3).

We would have hence both $X \to A \langle f_5, c_5 \rangle$ and $X \to A \langle f_7, c_7 \rangle$. Within the framework of NAL, the action in situations such as this one is clear: to choose the option with a higher level of confidence (see, e.g., Wang, 2013). Thus, the machinery of NARS gives criteria to know whether the steps of NSPP in a particular application authorize to come to the *Conclusion*, that is, the acceptance of the *Thesis*. Both an induction-deduction process and an analogy process (in which the similarity relation is a premise) are carried out. The conclusion with a higher value for c is selected. Then, the system reviews whether the selected conclusion has a value for f that allows accepting the *Thesis*, or the *Conclusion*.

One might argue that this does not completely remove the problem of proposals such as that of Schayer (1933). If the result of the inductive-deductive process were a statement with a significantly high value for c, the similarity relation would be irrelevant. I could respond that, as far as I am able to understand, NARS is designed in such a way that both inferences (induction-deduction and analogy) should be always made. Although the value of c_5 would be enough, c_7 would also be calculated to verify whether it is even higher. Similarity would always play a role.

Besides, Ganeri (2004) approach would help improve the result even more. If there is one more shared property (C) and the premises are these:

$$\{X \to B \langle f, c \rangle, X \to C \langle f, c \rangle, Y \to A \langle f, c \rangle, Y \to B \langle f, c \rangle, Y \to C \langle f, c \rangle, X \leftrightarrow Y \langle f, c \rangle\}$$

At a minimum, one more induction-deduction process would be possible. The rule of induction would allow this derivation:

$$\{Y \to A \langle f_1, c_1 \rangle, Y \to C \langle f_8, c_8 \rangle\} \vdash C \to A \langle f_9, c_9 \rangle$$

With $f_9 = f_1 = f_3$ and $c_9 = (f_8 \text{ x } c_1 \text{ x } c_8)/(f_8 \text{ x } c_1 \text{ x } c_8 + k)$.

Then the system would make this inference by virtue of the rule of deduction:

$$\{C \to A \langle f_9, c_9 \rangle, X \to C \langle f_{10}, c_{10} \rangle\} \vdash X \to A \langle f_{11}, c_{11} \rangle$$

Where $f_{11} = f_9 \text{ x } f_{10}$ and $c_{11} = f_9 \text{ x } c_9 \text{ x } f_{10} \text{ x } c_{10}$.

In this way, the values for confidence would be three: c_5, c_7, and c_{11}. The system would calculate the three values. The *thesis* would be admitted if *f* were high enough for the statement with the highest *c*. In that case, the result of NSPP would be that *x* has property *A*.

An outstanding point is the decision about what values for *f* and *c* would be high enough to admit the *Conclusion*. Beyond the fact that different values for constant *k* can mean different 'personalities' for the system, a decision must be made: from what frequency and confidence values is the *Thesis* acceptable? One might think that this decision must be human. Individual subjective aspects can have an influence on it. On the other hand, the context in which an inference with the structure of NSPP is proposed can be important, too. For example, we can think of a scientific context, or if preferred, a clinical context as that described by Ganeri (2004). That context would be different from other daily contexts (regarding this last point, the manner NARS can diagnose diseases can be interesting, see, e.g., Wang & Awan, 2011). In any case, today it is not hard to program any computer system to calculate, considering its own previous experience, optimal values. This should not be a problem for a system such as NARS.

Conclusions

NSPP is a particular kind of inference within Indian logic. It has received considerable attention from several points of view. Schayer tried to address it from classical logic. However, that account presents the difficulty that it ignores an essential component of NSPP: the similarity between the two elements.

The inference has been analyzed from a more psychological perspective, too. This analysis has tried to relate NSPP to the manner human beings think. The theory considered in this regard has been the theory of mental models. The theses of the latter theory that have been resorted to are those of possibilities and the way people derive inductive conclusions.

Ganeri's account refers to case-based reasoning. It has been understood that this proposal does not ignore similarity because it is linked to the idea of having one more shared property (not only *A* and *B*, but *C* as well).

Besides, it has been tried to show that Ganeri's idea can be useful both for the explanation based on the theory of mental models and for that of Schayer. In the first case, it justifies to a greater extent the election of one of the possibilities and the rejection of the other one. In the second case, it leads to a more complex formula with the structure of a Carnap's reduction sentence. That formula seems to include the similarity between elements because it deems *C* as an additional predicate.

My proposal has not been to raise any objections to any of these alternatives. I have attempted to explain that NAL can easily work with the formal structure of NSPP. This implies that a computer system such as NARS can also do that.

To argue in that direction, out of all the machinery of NAL, we only need two copulas, inheritance and similarity, and three inference rules, induction, deduction, and analogy.

Induction and deduction allow coming to the *Thesis*, *Ax*, from *Ay*, *By*, and *Bx*. This gives us values for *f* and *c*. On the other hand, the rule of analogy also enables to derive *Ax*, but from *Ay* and the similarity between *x* and *y*. This gives values for *f* and *c* as well. In addition, if we assume Ganeri's account, we can obtain even a third value both for *f* and *c*. We can also infer *Ax* from *Ay*, *Cy*, and *Cx*. This gives the system further information: one more value both for *f* and for *c*.

So, the only step left is for the system to decide whether it accepts the *Thesis* based on the values of *f* and *c*. Modifying the value of constant *k*, we can cause the system to accept or reject conclusions. But that appears to be different from deciding what the key numbers for *f* and *c* are. We need numbers (for *f* and *c*) indicating when the *Conclusion* should be considered and when it should be rejected. Subjective and contextual aspects can have an influence on the values of those numbers. However, it is not difficult for a computer system such as NARS to calculate them taking, for example, average values of previous accepted statements.

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